

Rupture Depth of an unreinforced concrete slab on grade?

In the development of standards for the aluminum patio enclosure design industry, this question has been frequently asked. Given an uplifting force at the outboard perimeter edge of an unreinforced concrete slab on grade as shown in Figure 1 we inquire at what distance from the uplifting force is the concrete slab likely to fail (rupture)?

Three sources indicate the approximate relationship between the modulus of rupture (R) for concrete and its compressive strength (f'_c):

f'_c = compressive strength of concrete in psi; and, R = modulus of rupture in psi; and, k = proportional constant (unitless):

ASTM C78 shows: $f'_c = R^2 / 100$;

Solving the equation from ASTM C78 for R, we get:

$$R = \sqrt{100 \cdot f'_c} \text{ or } R = 10 \cdot \sqrt{f'_c} \quad (k=10)$$

PCA (Portland Cement Association) shows a similar relationship:

$$R = 9 \cdot \sqrt{f'_c} \quad \text{where } k = 9 \text{ for rounded gravel and } 11 \text{ for crushed stone}$$

f'_c = compressive strength of concrete in psi

MR = modulus of rupture in psi

This relationship is expressed in ACI 318 as: $R = 7.5 \cdot \sqrt{f'_c}$

This last and most conservative value of **k = 7.5** will be used in the calculation.

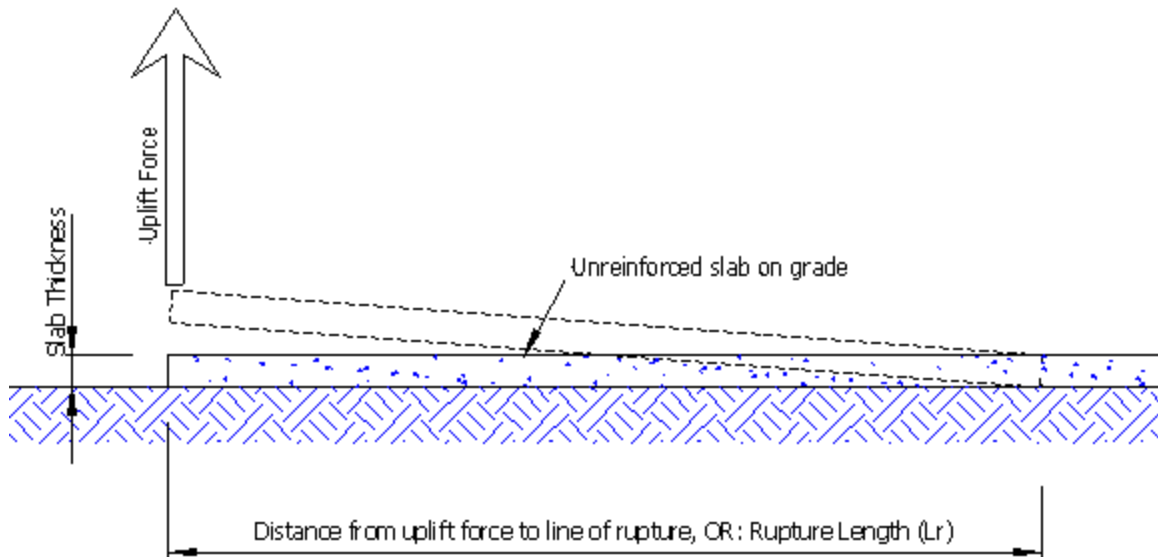


Figure 1: Slab on grade subjected to an uplift force due to wind load on a structure

Fracture Location on a Slab

Since the force uplifting the concrete must overcome the concrete mass, or, dead load of the concrete, we must know the Rupture location from the point of uplift (Length) and slab thickness to compute the uplift force resistance capacity of the concrete, and also include a rational factor of safety, FS.

The slab will be treated as a cantilever beam, one end (inboard) fixed, the other end (outboard) free. Hence, the applied bending moment at the point of rupture, or Rupture Depth will be expressed by the equation:

$$M_b = \frac{1}{2} \cdot \omega_D \cdot L_r^2 \quad (1)$$

where M_b is the applied bending moment, ω_D is the unit mass or weight of the concrete slab (beam) and L_r is the Rupture Length. The allowable bending moment for this section of concrete is expressed by the following equation:

$$M_a = F_b \cdot S_g \quad (2)$$

where, M_a is the allowable bending moment, F_b is the concrete's modulus of rupture, and S_g is the section modulus for the concrete strip (beam) as expressed by the equations:

$$F_b = \frac{f_r}{FS} \quad (3)$$

$$S_g = \frac{1}{6} \cdot A_g \cdot t \quad (4)$$

$$A_g = b \cdot t \quad (5)$$

$$\omega_D = \gamma_c \cdot A_g \quad (6)$$

γ_c is the weight density of the concrete slab.

Equation expressions 1 and 2 and solving for L_r we obtain the following expression:

$$L_r = \sqrt{\frac{2 \cdot M_a}{\omega_D}}$$

CONSTANTS:

Factor of Safety: $FS := \frac{1}{0.6}$

Constant with respect to Rupture Modulus $k := 7.5$

Fracture Location on a Slab

Example Problem

Geometry

$$\text{Slab width: } b := 12 \cdot \text{in}$$

$$\text{Slab Thickness: } t := 3.5 \cdot \text{in}$$

Material Properties

$$\text{Compressive Strength of Concrete: } f'_c := 2.5 \cdot \text{ksi}$$

$$\text{Weight Density of Concrete: } \gamma_c := 145 \cdot \frac{\text{lbf}}{\text{ft}^3}$$

Solution

$$\text{Gross (Net) Area: } A_g := b \cdot t$$

$$\text{Section Modulus: } S_g := \frac{1}{6} \cdot A_g \cdot t$$

$$\text{Dead load due to self-weight: } \omega_D := \gamma_c \cdot A_g$$

$$\text{Bending rupture stress: } f_r := k \cdot \sqrt{f'_c} \cdot \text{psi}^{0.5}$$

$$\text{Allowable bending stress of concrete: } F_b := \frac{f_r}{\text{FS}}$$

$$\text{Allowable bending moment for concrete: } M_a := F_b \cdot S_g$$

Since at Rupture Length ("L_r") M_a = M_b, then:

$$L_r := \sqrt{\frac{2 \cdot M_a}{\omega_D}} \quad L_r = 55.931 \text{ in}$$

Therefore, the uplift capacity per foot of concrete along outboard edge is

$$\text{Capacity}_{\text{Uplift}} := \omega_D \cdot L_r$$

$$\text{Capacity}_{\text{Uplift}} = 197.118 \text{ lbf}$$

The **ASD** (Allowable Stress Design) only applies a Factor of Safety to the material properties. The FS is required due to uncertainties from many sources such as material properties, geometric dimensions, loads, etc.